Formation and decay of a tributary-dammed lake, Laonong River, Taiwan

Hervé Capart1,*, Jammie P.C. Hsu1, Steven Y.J. Lai1, and Meng-Long Hsieh2

1 Department of Civil Engineering and Hydrotech Research Institute, National Taiwan University, Taipei 106, Taiwan (*) Corresponding author
2 Earth Dynamic System Research Center, National Cheng Kung University, Tainan, Taiwan

Abstract. We use field observations, laboratory experiments, and mathematical theory to characterize the full life cycle of a tributary-dammed lake. The natural lake formed in August 2007 at the confluence of the Laonong river, southern Taiwan, with a steep tributary. Due to heavy rains from Typhoons Wutip and Sepat, the tributary delivered massive amounts of sediment to the confluence. This caused the damming of the trunk river, the growth of a lake, and the demise of a bridge upstream of the confluence. The lake lasted into October 2007, gradually decaying due to incision and infill by subsequent stream flow. We reconstruct the episode using hydrological records, ground observations, and surveyed profiles. We then use the event to verify a mathematical theory of river and lake bed evolution. The theory is based on a diffusion description of river morphodynamics, constrained by backwater effects due to lake formation. After validation using laboratory experiments, the theory is applied to the Laonong field event. Good agreement is obtained between calculated and surveyed long profiles. The theory also explains the complex stage hydrograph recorded upstream of the confluence.
1. Introduction

In non-glaciated valleys, natural lakes most commonly form as a direct or indirect consequence of landslides [Hutchinson, 1957]. A landslide-dammed lake forms when a landslide directly slumps into a river [Costa and Schuster, 1988; Chen, 1999]. Landslides can also cause lake formation in a more indirect manner, by contributing large volumes of loose material to headwater streams. The products of mass wasting can then be mobilized by torrential or debris flows, and conveyed down tributary valleys to their confluences with trunk rivers [Benda et al., 2004; Lancaster and Casebeer, 2007]. A tributary-dammed lake forms when the resulting confluence aggradation is sufficiently rapid [Lane, 1955; Hsu and Capart, 2008]. Various contemporary river and lake profiles have been attributed to ancient tributary-damming episodes [Galay et al., 1983; Wright et al., 1998].

In a previous study [Hsu and Capart, 2008], we proposed a simple mathematical theory describing the response of alluvial rivers to sediment influx from tributaries. The theory is based on a diffusion description of river morphodynamics, constrained by backwater effects. The theory yields a criterion for the occurrence of tributary-dammed lakes, and analytical solutions for the co-evolving river and lake profiles. It predicts that a tributary-dammed lake will form if the sediment influx from the tributary exceeds twice the background bedload transit in the trunk river. Below this threshold, tributary sediment influx causes a cuspatate, symmetric aggradation of the trunk river, without forming a lake. Above the threshold, an asymmetric aggradation is predicted upstream and downstream of an expanding lake. This theory was checked quantitatively against idealized laboratory experiments [Hsu, 2007]. Field verification, however, could not be performed due to the lack of adequate data.

In August 2007, heavy rains brought by Typhoons Wutip and Sepat triggered sustained debris flows in a steep tributary of the Laonong River, southern Taiwan. The resulting massive influx
of sediment formed a dam at the tributary confluence, causing a lake to grow and drown the
upstream reaches of the trunk river (Fig. 1). The lake lasted into October 2007, gradually
decaying due to subsequent stream flow. We were able to observe the lake before its decay. It
was also possible to collect unusually complete data regarding the whole life cycle of the lake,
including measurements acquired by the Taiwan Water Resources Agency (WRA) and a field
survey we conducted in February 2008. The data include hydrological records, and
topographic transects acquired before and after the lake formation episode. The event thus
provides a unique opportunity to verify our mathematical theory of river and lake evolution.

To simulate the Laonong episode, it is necessary to extend the theory beyond the analytical
model proposed in *Hsu and Capart* [2008]. In field conditions, more complicated
circumstances must be taken into account, including the highly unsteady river discharge
associated with typhoon floods. Because the 2007 Laonong episode involved the full life
cycle of a tributary-dammed lake, our model of lake initiation and growth must also be
extended to simulate lake decay and disappearance. As a result, the governing equations of the
theory need to be solved numerically instead of analytically.

In this paper, we summarize our observations of the 2007 tributary-dammed lake. We present
the data analysis performed to reconstruct the evolution of the river bed and water level at
different stages of the lake life cycle. Next, we explain the bases of our theory, and how
solutions are calculated. To check the numerical model, we compare model results with
analytical solutions (for lake growth) and laboratory experiments (for the full lake life cycle).
Finally, we apply the model to the Laonong River and verify that the theory can reproduce the
observed river and lake evolution. This suggests a role for simplified, but laboratory-tested
mathematical theory in quantitative investigations of fluvial processes in the field.
2. Study site and lake observations

2.1. Setting

The south-flowing Laonong River, with a watershed area of ~2000 km², is a major tributary of the Kaoping River that drains mountains up to 4000 m high in southwestern Taiwan (Fig. 2). Except for its lowest 20-km long course that flows on the Pintung plain, the Laonong River runs between the main divide of the Central Range (the backbone range of Taiwan Island) to the east and the secondary Yu-shan Range to the west. The lower half of this mountain river course flows roughly along a fault zone separating the metamorphosed rock formations (argillite, slate, meta-sandstone) underlying the Central Range from the sandstone/shale formations constituting the Yu-shan Range. This fault-bounded river reach is relatively straight, and flanked by a series of river terraces. As the river reach is also relatively wide and gentle, it provides optimal locations for the formation of tributary fans. Almost all tributaries joining this trunk river reach develop alluvial or debris fans. Most of them, however, have been abandoned and incised by the trunk river, forming fan terraces of various sizes/heights. These fan terraces, up to 200 m high above the trunk river, document episodic activities of landslides/debris flows in tributary watersheds and are among the most spectacular landforms exhibited in Taiwan’s mountains [Lin, 1957; Hsieh and Chyi, 2010]

The Laonong Watershed experiences a tropical-subtropical monsoon climate, receiving annual rainfall of 2000–4000 mm. The resulting mean annual discharge at the Laonong gauging station, where the river emanates from mountains, is approximately 60 m³ s⁻¹. Rainfall comes mainly from summer monsoon and tropical typhoons and shows strong seasonality. At the Laonong gauging station (Fig. 2B), mean monthly discharges fall in the ranges 50–170 and 15–40 m³ s⁻¹ during the wet (May to October) and dry seasons, respectively [Shiau and Wu, 2009]. Individual flood peaks associated with typhoon events can reach daily discharges as high as 1000 m³ s⁻¹.
The active channels and floodplains characterizing the mountain course of the Laonong River are confined laterally by bedrock and, occasionally, by sedimentary fluvial or fan terraces. Bedrock, however, is rarely exposed along the bed of the lower, fault-bounded portion of the Laonong River, even during low-flow conditions. Instead, the bedrock floor of this relatively wide and gentle river is buried under alluvial cover. Drilling performed by the Water Resources Agency (WRA) three kilometers downstream of our study site reveals bedrock at a depth of 10 to 20 m beneath the alluvium. The current lower mountain portion of the Laonong River may thus be considered semi-alluvial [Brooks and Lawrence, 1998]: valley sides are confined mainly by bedrock, but the valley bottom consists of alluvial fill of significant thickness.

Among the tributaries of the Laonong River, the most prone to episodic landslides and debris flows is probably the Pu-tun-pu-nas River (Fig. 1). This is a steep tributary (relief: 1500 m; mean gradient: 0.3) draining the Yu-shan Range. It has a drainage area of 6.2 km², two orders of magnitude smaller than the contributing area of 542 km² of the trunk river upstream from their confluence. Although it contributes only a very small fraction of the total water discharge, this tributary constitutes a major source of sediment supply to the Laonong River. Ancient sediment supply episodes are manifested by elevated fan terraces surrounding the confluence, for which radiocarbon dating indicates ages between 3000 and 7000 years BP [Hsieh and Chyi, 2010]. Since the 1980s, we have observed repeated debris flow activity in this tributary, causing the episodic growth and incision of the debris fan at its mouth (Fig. 3A). One such episode caused the formation of the tributary-dammed lake in August 2007. More recently, an even stronger alluviation event occurred at this location, triggered by the record-setting rainfall brought by Typhoon Morakot, in August 2009. Based on WRA survey data, this event caused 20 meters of river bed aggradation three kilometers downstream of the
Pu-tun-pu-nas confluence. The 2007 lake formation episode thus represents a remarkable, but not exceptional event for this tributary confluence.

River bed changes downstream of the Pu-tun-pu-nas confluence are of engineering concern because of a major water resources scheme that is currently being implemented by the WRA. Called the Tseng-Wen Reservoir Transbasin Diversion Project, the scheme is to divert part of the water discharge of the Laonong River to the Tseng-Wen Reservoir, two valleys away (Fig. 2B). The project involves two tunnels pierced through mountain ridges, an aqueduct, and a weir across the Laonong River. The site of this weir, not yet built, is located only 3 km downstream of the Pu-tun-pu-nas confluence (Fig. 2C). Tunneling works were under way, and the overall scheme was scheduled for completion before 2012 [Yang et al., 2009]. Delays are likely, however, following the major alluviation event of August 2009, which buried the Laonong tunnel entrance under a thick cover of sediment. In retrospect, the August 2007 lake formation episode can be seen as a precursor event, highlighting the potential of local tributaries to supply massive amounts of sediment to the trunk river.

2.2. Lake observations

We first observed the tributary-dammed lake at the Pu-tun-pu-nas confluence on September 26, 2007, one month after the passage of Typhoons Wutip (August 8–9) and Sepat (August 16–19) and a week after Typhoon Wipha (September 17–19) (Fig. 3B). At that time, the trunk river had already re-incised the fan by several meters (Fig. 4A), but the lake had not yet vanished (Fig. 3C). The lake drowned the trunk river upstream of the confluence over a length of approximately 800 m. We re-visited the site on January 6, 2008, three months after the passage of Typhoon Krosa (October 4–7), the last typhoon of the year 2007 (Fig. 3D). By this time, the lake was entirely eliminated, and the re-incision of the fan by the trunk river had left a 20-m high terrace riser bounding the abandoned fan (Fig. 4B). Upstream of this newly
formed fan terrace, a flight of fluvial terraces up to 8 m in height was observed along the trunk river. These terraces are widely capped by 1–2 m thick sands, interpreted to have accumulated along sheltered sides of the flood channels during the episode.

The lake formation episode caused the destruction of a bridge, the Hsing-huei Bridge, built across the Laonong River 1 km upstream of the Pu-tun-pu-nas confluence (Fig. 2C). Constructed in 1995, this road bridge had a span of approximately 100 m. On September 26, we observed that the bridge deck had been swept away, leaving its abutments and central pile intact (Fig. 3E–F). The deck failure appears to have involved buoyant uplift and horizontal drag by flood waters, similar to the failure mode of various coastal bridges destroyed by Hurricane Katrina [Chen et al., 2009]. In 2005, the WRA instrumented the Hsing-huei Bridge with an automated stage measurement device. This stopped operating on August 19, 2007, after recording a rapid water rise up to the level of the bridge deck [Water Resources Agency, 2007]. We believe that this rise in level ensued from the growth of the tributary-dammed lake. The timing of the bridge destruction thus indicates that it is mainly Typhoons Wutip and Sepat, rather than Typhoon Wipha, which caused the aggradation of the Pu-tun-pu-nas debris fan and resultant lake formation.

Except for incision and trimming by both the trunk and tributary channels, we do not observe significant changes in topography of the Pu-tun-pu-nas debris fan from September 26, 2007 to January 6, 2008 (Fig. 4). This implies that the amount of sediment supplied by the Pu-tun-pu-nas River during Typhoon Krosa was relatively small, compared to that supplied during Typhoons Wutip and Sepat. Still, flood waters generated during Typhoon Krosa were high enough to undermine the abutment of a 12-year-old suspension bridge located 2 km upstream of the Pu-tun-pu-nas River junction. To facilitate reference, the key dates mentioned in our account of the river and lake evolution are listed in Table 1.
3. Data collection and analysis

3.1. Topographic survey data

Topographic surveys of the river bed were performed both before and after the 2007 episode of lake formation and decay (Fig. 5). The first survey was performed by the Water Resources Agency (WRA) in August 2004, one month after Typhoon Mindulle (June 28–July 03). The WRA survey covers the trunk river only, from 1 km upstream to 5 km downstream of the confluence, and includes 51 regularly spaced cross-sections. We performed the second survey in February 2008, during the low flow period. This survey includes long profiles of the water line and tributary terrace riser, supplemented by 12 cross sections of the trunk river and tributary fan. Along sheltered sides of the Laonong River, some cross-sections feature thick sand-capped bars, which we interpret as slackwater deposits [Jones et al., 2001; Bohorquez and Darby, 2008]. Their elevations are taken as indicators of the water stage reached during the high flow preceding our survey (Typhoon Krosa flood). To enable retrieval of elevation data from past photographs, we further surveyed recognizable feature-points that could serve as controls for photo calibration. We used this approach to retrieve water surface data from 5 ground photographs acquired on September 26, 2007, when the lake was still present.

The results of our survey are linked to those of the WRA through a common benchmark. This was installed in July 2004 by the WRA on the right abutment of Hsing-huei Bridge, and fortunately was not washed away by the flood. To convert survey data to long and transverse profiles, we constructed a system of valley axes and transects, following the approach of Capart et al. [2007]. This system is used to define along- and across-valley coordinates (x, y) for the Laonong river, and (x’, y’) for the Pu-tun-pu-nas tributary, with a common origin positioned at the confluence (Fig. 5A).
3.2. Streamflow data

The Laonong River discharge is recorded at two gauging stations of the WRA: Laonong Station (station number 1730H031), located 24 km downstream of Hsing-huei Bridge, and Achiba Bridge (station number 1730H044), 8 km upstream. A continuous stage record is available at Hsing-huei Bridge from January 1st, 2005 to August 19, 2007. Direct measurements of discharge were also conducted 2 or 3 times per month at Hsing-huei Bridge, until the bridge was destroyed (Fig. 6).

Discharge measurements at Hsing-huei Bridge show no clear relation with simultaneous stage measurements at the same site (Fig. 7A). Even for the period covered by the data, it is therefore not feasible to estimate the daily flow rate at Hsing-huei Bridge from the measured daily stage data. The discharge at the bridge, however, correlates well with simultaneous discharge records at Laonong Station and Achiba Bridge. A daily streamflow signal at Hsing-huei Bridge can thus be derived from these records, considering that discharge at a site is proportional to its contributing watershed area. This yields estimates:

\[
Q_1(t) \approx \frac{A}{A_u} Q_u(t), \quad Q_2(t) \approx \frac{A}{A_d} Q_d(t).
\]  

Here \(Q_u(t)\) is the daily discharge at the upstream station (contributing area \(A_u = 404 \text{ km}^2\)), \(Q_d(t)\) is the daily discharge at the downstream station (contributing area \(A_d = 812 \text{ km}^2\)), and \(A = 542 \text{ km}^2\) is the contributing area at Hsing-huei Bridge. In practice, we average \(Q_i(t)\) and \(Q_j(t)\), except for two periods during which data from one of the two stations are inconsistent with the partial measurements at Hsing-huei Bridge. The resulting reconstructed daily streamflow record is plotted in Fig. 6, together with the discharge measurements at the bridge. The estimated and measured flow rates generally agree, and the reconstruction allows us to extend the discharge record at Hsing-huei Bridge to periods of no direct measurements.
3.3. Stage-discharge relation at Hsing-huei Bridge

The reconstructed daily streamflow at Hsing-huei Bridge can be used to re-examine the relation between stage and discharge at that location. In Fig. 7B, we plot the measured stage record at the bridge against the discharge estimated by Eq. 1, raised to the power 2/3. Like the partial data of Fig. 7A, the daily data of Fig. 7B do not fall on a single rating curve. Short term variations, however, trace mutually parallel curve segments consistent with the law of parallelism of ratings [Freeman and Bolster, 1910; see Schmidt and Yen, 2008]. These segments conform to the relation

\[ z_w = aQ^{2/3} + b = a Q^{2/3} + z_s \]  

(2)

with the same slope \( a \approx 0.065 \) for all segments, but different offsets \( b \), associated with changes in river sediment bed elevation \( z_s \). We use this observation to obtain a depth-discharge relationship. Assuming that the sediment bed elevation evolves more slowly than either the discharge or the water stage, we approximate the instantaneous bed elevation at a given day \( t \) by its average over the previous week,

\[ \bar{z}_s(t) = \frac{1}{7} \int_{t-8}^{t-1} (z_w - a Q^{2/3}) dt , \]  

(3)

where time is expressed in days. We can then estimate the water depth on day \( t \) from

\[ h(t) = z_w(t) - \bar{z}_s(t) , \]  

(4)

constructed without using the discharge \( Q(t) \) on day \( t \). Figure 7C shows the depth-discharge relation obtained in this way, plotting the depth \( h(t) \) estimated from Equations 3 and 4 against the daily discharge \( Q(t) \) from Eq. 1. As expected, the data points approximately collapse onto a rating curve of the assumed form. This confirms that the water level at Hsing-huei Bridge generally rises and falls under the influence of two contributions: fast variations due to transient streamflow \( Q(t) \), and slow variations due to changes in local sediment bed elevation \( z_s(t) \). The plot also highlights an exception to this rule, on the last day of data acquisition.
prior to the submersion of Hsing-huei Bridge. On August 19, 2007, the water level attained a
record high, reaching the bridge deck, in a way that cannot be accounted for by the rating
curve. On its own, the flowrate $Q(t)$ on that day was not sufficient to raise the water depth so
high. Instead, the bridge was likely submerged by the backwater profile associated with the
aggrading tributary dam, 1 km downstream.

4. Mathematical description

4.1. Assumptions and governing equations

Our aim is to predict the evolution of the long profile of a trunk river subject to strong
sediment influx from a tributary. Let $z_s(x, t)$ denote the elevation of the trunk river above a
reference datum. The river at time $t_0$ is taken to start from a known sediment bed profile
$z_s(x, t_0)$, where $x$ is the curvilinear coordinate measured along the valley axis in the direction
of river flow. For simplicity, we place the origin $x = 0$ at the tributary confluence. We
neglect the water flux discharged from the tributary and consider only its sediment
contribution, $I(t)$, the volumetric flux of bed material (sediment + pore space) dumped into the
trunk river. We further assume that the trunk river flows in a channel of constant width $B$.

Under these conditions, the equation governing the evolution of the sediment bed profile
$z_s(x, t)$ is the Exner equation

$$B \frac{\partial z_s}{\partial t} + \frac{\partial J}{\partial x} = I(t) \delta(x),$$  \hspace{1cm} (5)

where $J(x, t)$ is the bedload transport rate in the trunk river, defined as a flux of bed material
volume (sediment + pore space) across a given cross-section. The Dirac delta $\delta(x)$ on the
right-hand side is used to represent the tributary sediment influx as a point source of
time-varying strength $I(t)$ at the origin.

In addition to the sediment bed profile $z_s(x, t)$, we also simulate the water surface profile
$z_w(x,t)$ along the trunk river. We assume that the unsteady water discharge $Q(t)$ varies sufficiently slowly that it can be considered uniform along the valley segment of interest. Accordingly, the water surface is taken to adjust in a quasi-steady manner to the evolution of the sediment bed. Following Hsu and Capart [2008], we subdivide the river reaches into two types: running-water reaches, where uniform flow parallel to the sediment bed is assumed, and standing-water reaches, where an approximately horizontal backwater profile is forced from downstream. At each location $x$, the water surface must satisfy the following system of complementary inequalities:

$$z_w - z_s \geq h_0, \quad \frac{\partial z_w}{\partial x} \leq 0, \quad (z_w - z_s - h_0) \frac{\partial z_w}{\partial x} = 0,$$  \hspace{1cm} (6)

where $h_0$ is the normal depth of the flow in reaches of running water. In the above system, the first inequality states that the water depth $z_w - z_s$ must either coincide (in running-water reaches) or exceed the normal depth (in standing-water reaches). The second inequality requires the water surface to either tilt down-valley (in running-water reaches) or approach the horizontal (in standing-water reaches). The last condition holds that at any location $x$ one or the other of the inequalities must reduce to an equality, insuring that the reach is characterized by either running or standing water. Taken as a whole, the system forces the water surface to decrease monotonously from upstream to downstream, even if the sediment bed profile does not. Because the sediment bed $z_s(x,t)$ evolves in time, the subdivision between running and standing-water reaches is not fixed once and for all, but continuously adjusts to the topography.

To simplify matters further, we assume that the normal depth $h_0$ depends only on the instantaneous discharge $Q(t)$, as specified by the depth-discharge relation

$$h_0(x,t) = h_0(Q(t)) = \left( \frac{Q(t)}{CBg^{1/2}} \right)^{2/3}.$$  \hspace{1cm} (7)
where $C$ is a dimensionless rating curve coefficient assumed constant along the valley segment considered, and $g$ is the gravitational acceleration. With the equivalence $a = (CBg^{1/2})^{-2/3}$, this is simply a dimensionally correct version of the empirical depth-discharge relation of Eq. 2. The above description may appear a drastic over-simplification of the standard theory of open-channel flow [see e.g. Henderson, 1966]. In the following sections, however, we will show that the resulting predictions agree well with both experimental and field data.

We finally link the bedload transport rate $J(x,t)$ to the prevailing river bed and flow conditions. Following Hsu and Capart [2008], we adopt the following modified stream power law for bedload transport:

$$J(x,t) = KQ(t) \max \left\{ -\frac{\partial z_w}{\partial x} - S_{\text{min}}, 0 \right\}, \quad (8)$$

where $K$ is a dimensionless transport coefficient, and $S_{\text{min}}$ is the slope threshold proposed by Mitchell [2006; see also Lai and Capart, 2007], below which transport is taken not to occur. In the above relation, the gradient term driving sediment transport is provided by the water surface slope $-\partial z_w / \partial x$, not the sediment bed slope $-\partial z_s / \partial x$. This choice is made in order to suppress bedload transport in reaches of standing water, where $\partial z_w / \partial x = 0$. When the slope threshold is set to zero ($S_{\text{min}} = 0$), and where $S = -\partial z_s / \partial x = -\partial z_w / \partial x$ (in reaches of running water), the relation reduces to the classical stream power relation $J \propto QS$ proposed by Lane [1955]. When only reaches of running water are present (even when $S_{\text{min}} \neq 0$), substitution of the above transport relation into the Exner equation (Eq. 5) yields the linear diffusion equation with source term

$$B \frac{\partial z_w}{\partial t} - KQ(t) \frac{\partial^2 z_w}{\partial x^2} = I(t)\delta(x), \quad (9)$$

with a time-varying diffusivity $D(t) = KQ(t) / B$. When reaches of standing water are present, transport is locally suppressed, making the diffusivity non-uniform and the problem
non-linear. For steady water discharge without tributary influx, Eq. 10 further reduces to the linear diffusion model proposed by Begin et al. [1981].

In summary, the above theory requires information about the valley segment to set the channel width $B$, initial conditions $z_i(x,t_0)$ and boundary conditions upstream and downstream. The river discharge $Q(t)$ and tributary sediment supply $I(t)$ must also be prescribed. Finally, one must select appropriate values for the following dimensionless constitutive coefficients: $K$ (bedload transport coefficient), $S_{\text{min}}$ (bedload transport slope threshold), and $C$ (depth-discharge rating curve coefficient). These coefficients can either be calculated on the basis of more detailed descriptions of water flow and sediment transport, or calibrated from available experimental or field data. In what follows, we will adopt the second approach and check that the above data requirements can be met, even for a field case where only partial information is available.

4.2. Numerical scheme

Under special circumstances, it was shown in Hsu and Capart [2008] that the above equations admit self-similar analytical solutions. In particular, such solutions can describe the growth of a tributary-dammed lake due to a steady sediment influx from a tributary to a channel of constant initial slope. In the present study, however, we are interested in more general conditions for which analytical solutions are insufficient. To describe the full life cycle of a tributary-dammed lake, solutions must be constructed by numerical means. For this purpose, we set up the following finite volume discretization. A finite domain $x_U \leq x \leq x_D$ is assumed, with upstream and downstream boundaries $x_U, x_D$, inside which the sediment bed and water surface elevations $z_i(x_i), z_w(x_i)$ are sampled at equally spaced positions

$$x_i = x_U + (i - \frac{1}{2})\Delta x,$$

where $i = 1,...,m$ and $\Delta x = (x_D - x_U)/m$. At any given time $t$, the water surface profile
$z_w(x,t)$ can be found from the sediment bed profile $z_s(x,t)$ using the solution

$$z_w(x,t) = \max \{z_s(\xi,t) + h_0(Q(t)) \text{ for } \xi \geq x \}$$  \hspace{1cm} (11)\]

constructed to satisfy the complementary inequalities of Eq. 6. For a discrete profile $z_s(x_i)$, this translates into the following very simple algorithm. Assuming a given water level $z_w(x_m)$ at the last grid point, the full water profile $z_w(x_i)$ is obtained by iterating the statement

$$z_w(x_i) = \max \{z_w(x_{i+1}), z_s(x_i) + h_0 \}, \hspace{1cm} i = m-1, m-2, ... , 1$$ \hspace{1cm} (12)

which represents a one-way sweep from downstream to upstream. Once the water surface profile is known, the sediment bed profile at the next time step can be obtained by integrating the Exner equation (Eq. 5). The sediment bed level $z_s(x_i)$ is advanced from time $t_k$ to time $t_{k+1} = t_k + \Delta t$ using the explicit finite volume statement

$$z_s(x_i, t_{k+1}) = z_s(x_i, t_k) + \frac{\Delta t}{B \Delta x} \{J_{i-1/2} - J_{i+1/2} \} + \frac{\Delta t}{B \Delta x} I_i.$$ \hspace{1cm} (13)

The second term on the right-hand-side represents the balance of bedload fluxes across the upstream and downstream faces of each finite volume. These bedload fluxes $J_{i+1/2}$ are calculated using the following discrete version of the modified stream power law (Eq. 8)

$$J_{i+1/2} = KQ \max \left\{ -\frac{z_w(x_{i+1}) - z_w(x_i)}{\Delta x}, 0 \right\},$$ \hspace{1cm} (14)

where all variables are sampled at time $t_k$. The third term on the right-hand-side of Eq. 13 is the contribution from the tributary sediment influx, given by

$$I_i = \begin{cases} I(t_k), & \text{if } x_i - \Delta x/2 \leq 0 < x_i + \Delta x/2, \\ 0, & \text{otherwise}. \end{cases} \hspace{1cm} (15)$$

Note the mass conservation property of the finite volume statement (Eq. 13). If the tributary source term is turned off ($I(t) = 0$) and no sediment enters through the domain boundaries, sediment can be transferred between cells but the total sediment volume $\sum_i z_s(x_i) B \Delta x$ will not vary. Because the rate-setting process is a diffusion equation, with diffusivity $D = KQ / B$, the stability of the explicit scheme is subject to the condition
\[ \Delta t < \frac{1}{2} \frac{(\Delta x)^2}{D}. \]  

For the calculations presented below, we start from piecewise linear initial conditions

\[ z_s(x, t_0) = \begin{cases} 
  z_0 - S_U x, & x_U < x \leq 0, \\ 
  z_0 - S_D x, & 0 \leq x < x_D, 
\end{cases} \]  

where \( S_U \) and \( S_D \) are upstream and downstream initial slopes. We further send the boundaries \( x_U \) and \( x_D \) sufficiently far upstream and downstream of the valley segment of interest to be able to assume the following running water, constant flux conditions at the boundaries

\[ J_{l/2} = J(x_U) = KQ \max\{S_U - S_{\min}, 0\}, \]  

\[ J_{m+l/2} = J(x_D) = KQ \max\{S_D - S_{\min}, 0\}, \]  

\[ z_u(x_m, t_k) = z_s(x_m, t_k) + h_b(Q(t_k)). \]

With these initial and boundary conditions, the numerical scheme is now completely specified. We present a calculation example in the next section.

### 4.3. Calculation example

To illustrate the theoretical and numerical description, we consider the following conditions, corresponding to one of the experimental runs of Hsu [2007]. The narrow channel of width \( B = 1 \) cm starts from an alluvial bed of constant inclination \( S_U = S_D = S_0 = 0.11 \), in equilibrium with the water discharge \( Q = 4.67 \) ml s\(^{-1}\) and background sediment flux \( J_0 = 0.256 \) cm\(^3\) s\(^{-1}\) supplied upstream of the channel, both held steady throughout. The tributary sediment influx follows the simple history

\[ I(t) = \begin{cases} 
  0, & t < t_1, \\
  I_1, & t_1 \leq t < t_2, \\
  0, & t_2 \leq t. 
\end{cases} \]  

The influx at location \( x = 0 \) is started at time \( t_1 = 0 \) s, held constant at value \( I_1 = 1.13 \) cm\(^3\) s\(^{-1}\)
over time interval $t_1 \leq t < t_2$, and terminated at time $t_2 = 239$ s. Because the water discharge is held steady in this case, a single value $h_0 = 2$ mm is adopted for the normal flow depth, corresponding to the experimentally observed water depth over the initial constant slope bed. Separate measurements of bedload transport $J$ in steady uniform flow for various discharges $Q$ and channel slopes $S = -\partial z_s / \partial x = -\partial z_w / \partial x$ were performed by Hsu [2007] to characterize the bedload transport relationship $J(Q, S)$ for the conditions of her experiments. These data were used to determine best fit values $K = 1.66$ and $S_{\text{min}} = 0.077$ for the transport coefficients (see Table 2 for a summary of the input and output parameters). Numerical computations are performed with upstream and downstream boundaries at locations $x_u = -200$ cm, $x_d = 200$ cm, spatial step 5 mm, and time step $\Delta t = 0.01$ s.

Computed results are presented in Fig. 8. The first panel (Fig. 8A) shows the evolution of the channel during the period $t_1 \leq t < t_2$ when the tributary sediment flux is active. The corresponding strong sediment influx at the origin $x = 0$ forces the formation and growth of a tributary-dammed lake. Fed by the tributary, aggradation of the main channel occurs downstream of the tributary junction, creating a deposit of maximum thickness at the junction itself. This generates a dam crest of rising elevation, upstream of which ponds a lake of rising stage and expanding length. The upstream transgression of the lake gradually drowns thin deposits laid down by the upstream river. Over this evolving sediment bed profile, the water surface is parallel to the bed profile upstream and downstream of the lake, corresponding to reaches of running water, and horizontal in the zone of standing water associated with the lake, where sediment transport is suppressed. Because the dam aggradation is gradual, the water flow across the lake and past the dam crest is never interrupted, and the formation of the lake blocks sediment transit only.

For the period of dam and lake growth covered by Fig. 8A, neither the initial conditions (a
linear profile) nor the sediment influx (a point source at the origin) impose any externally prescribed length scale on the problem. This allows the sediment bed profile to respond in the geometrically self-similar fashion

$$\frac{z_s}{\sqrt{S_0 J_0(t-t_i)}} = f\left(\frac{x}{\sqrt{J_0(t-t_i)/S_0}}\right).$$  \hspace{1cm} (22)

In Hsu and Capart [2008], we exploited this special characteristic to derive the similarity function $f(\cdot)$ in analytical form. This function is defined piecewise by the following expressions

$$f(\xi) = \begin{cases} 
-\xi + \frac{2\mu}{\text{erfc}(\frac{1}{2} \lambda)} \text{erf}(-\frac{1}{2} \mu \xi), & \xi \leq -\lambda / \mu, \\
-2(I_1/J_0 - 1)\mu^2 \xi / (\sqrt{\pi} \lambda), & -\lambda / \mu \leq \xi < 0, \\
-\xi + 2(I_1/J_0 - 1) \mu \text{erfc}(\frac{1}{2} \mu \xi), & 0 \leq \xi,
\end{cases}$$  \hspace{1cm} (23)

where $\mu = \sqrt{1 - S_{\text{min}}/S_0}$, and function $\text{erf}(\xi) = \exp(-\xi^2) / \sqrt{\pi} - \xi \text{erfc}(\xi)$ is the first integral of the complementary error function $\text{erfc}(\xi) = 1 - \text{erf}(\xi)$ [see Carslaw and Jaeger, 1959]. The parameter $\lambda$ is the root of a transcendental equation taking numerical value $\lambda = 0.893$ for the conditions of this particular run ($S_{\text{min}}/S_0 = 0.7$, $I_1/J_0 = 4.4$). To check the numerical scheme, Fig. 8B compares the computed profiles with the analytical solution. Excellent agreement is recorded: plotted in normalized coordinates, the numerical sediment bed profiles obtained for different times $t = 0$ to 239 s very nearly coincide, collapsing onto the analytical curve of Eq. 23.

Because lake decay starts from generic conditions, inherited from the tributary-damming process, the analytical solutions derived by Capart et al. [2007] for lake breaching and infill do not apply. Numerical computations can proceed, however, and the resulting profiles are shown in Fig. 8C. They cover the period $t_2 \leq t \leq t_1$ between the termination of the tributary sediment supply and the complete elimination of the lake. Upon termination of the sediment
supply, at time $t_2 = 239$ s, the lake decays due to two simultaneous processes. Downstream, erosion of the river bed occurs, leading to the gradual incision of the lake outlet and associated drainage of the lake. Upstream, delta progradation into the shrinking lake leads to infill of the lake depression. The lake is eliminated when the combined infill and incision shrink the lake to a point. After upstream and downstream reaches merge back together, the reconnected alluvial channel continues to evolve under the action of running water. The time $t_3$ at which the lake disappears is not prescribed by the external conditions, but computed as part of the solution. In this particular case, the value obtained for the time of lake elimination is $t_3 = 379$ s. Qualitatively, the lake formation and decay processes presented in Fig. 8 agree with descriptions by Lane [1955] and Holmes [1945; p. 155-156 and Fig. 66]. Quantitative comparison with laboratory and field profiles are presented in the next sections.

5. **Comparison with experiments**

To test the theory and computations, we first compare calculated results with the laboratory experiments of Hsu [2007]. These experiments were used in Hsu and Capart [2008] to validate analytical results for lake onset and growth. Here we use them to validate more general numerical solutions for both the growth and decay of a tributary-dammed lake. The experiments were conducted using the set-up shown in Fig. 9. A long, narrow flume (length = 250 cm, width = 1 cm) is inclined at a slope $S_0 = 0.11$ (determined using a leveling tube). Upstream, the flume is supplied with a steady water discharge from a constant head tank, and with a steady flux of sediment from a silo of dry sand. To facilitate control of the sediment flux, the sand does not freely stream down from the bottom outlet of the silo. It is instead entrained laterally by a motor-driven conveyor belt of adjustable speed (apron feeder). Inspired from a similar device at National Central University [Hsiau et al., 2004], this sand-feeding mechanism allows precise fine-tuning of the sediment flux simply by varying the belt speed. The mechanism is used to adjust the upstream sediment flux to the prescribed
slope and water discharge, until equilibrium transport conditions are achieved, over a sand
deposit of uniform thickness. A second sand silo is used to feed a prescribed influx of dry
sand to the middle of the flume. This supply can be activated and de-activated at prescribed
times to represent intermittent sediment influx from the tributary. Sand of median diameter
\( d_{50} = 0.32 \text{ mm} \) and coefficient of uniformity \( d_{60} / d_{10} = 1.84 \) is used as sediment material.

Selected snapshots for one experimental run are presented in Fig. 10. The response of the
channel is observed through the transparent side wall of the flume. Lighting is provided by
back-illumination through a translucent white panel placed behind the other side wall, and the
channel evolution is recorded by time lapse photography. The photographs are used to retrieve
sediment bed and water surface profiles at successive times, via a calibrated transformation
from image to metric coordinates. The conditions for this run are those adopted earlier for the
calculation example (see Section 4.3 and Table 2).

The resulting profiles are shown in Fig. 11. Overall, the observed response matches that
predicted by the theory. For such strong tributary sediment influx \(( I_1 / J_0 > 2 )\), a
tributary-dammed lake grows during the period of tributary activity \( t_1 < t < t_2 \), then gradually
decays once the tributary sediment influx stops. Lake decay results from two simultaneous
processes: infill by a prograding delta at the lake inlet, and incision of the dam crest at the
lake outlet. The lake is eliminated once the prograding delta front reaches the receding dam
crest. For this experimental run, the complete elimination of the lake occurs at time \( t_3 = 387 \text{ s} \), which compares well with the predicted time \( t_3 = 379 \text{ s} \) obtained from the numerical
computations.

Despite the overall agreement, minor differences between the simulated (Fig. 8) and observed
profiles (Fig. 11) can be noted. The main discrepancies concern the upstream-facing slope of
the tributary dam and the downstream-facing foreset of the prograding delta. Both slopes are
assumed vertical in the theory, but relax to the angle of repose of the sediment material
($\varphi = 36^\circ$) in the experiments. The influence of this finite angle of repose could be introduced
into the theory [Voller et al., 2004; Lai and Capart, 2009], but this would complicate the
numerical scheme considerably. For this reason, we assume like Voller et al. [2006] that the
angle of repose is sufficiently steep relative to the riverbed inclination to be approximated by
a vertical (in the analytical solutions), or near-vertical segment (spread over a few grid points
in the numerical computations). Also, instead of the sharp dam crests produced in the
simulations, more rounded crests are observed in the experiments. This can be attributed to
pluviation over a finite width (instead of a point source), and to deviation of the experimental
flow from the abrupt transition between horizontal backwater and bed-parallel uniform flow
assumed by the theory.

To make the comparison quantitative, we plot in Fig. 12 the computed and measured time
histories of the bed and water elevation at selected cross sections. The bed elevations shown
in Fig. 12A exhibit distinct responses upstream and downstream of the tributary confluence.
At downstream cross sections, the bed evolution tracks the rise and fall of the tributary dam
crest, with some delay and attenuation. At upstream cross sections, the bed evolution is more
complex due to the influence of the expanding and shrinking lake. Upstream bed elevations
within reach of the lake rapidly rise at first, freeze at a constant level when drowned by the
lake, then undergo a step-like jump upon passage of the prograding delta associated with lake
decay. Cross sections further up valley, never reached by the lake, evolve more gradually
under the influence of the wedge-like aggradation upstream of the lake.
Water surface elevations depicted in Fig. 12C undergo similar evolutions. Because the channel evolves gradually, the water surface evolution is slaved to the sediment bed evolution, as assumed by the theory. At most locations, the water stage rises along with the local bed height. The key exception occurs when a cross section is drowned by the lake, in which case non-local effects intervene. The water stage is no longer determined by the local bed elevation (increased by the running water depth), but instead rises and falls in lockstep with the water stage at the lake outlet, itself determined by the elevation of the tributary dam crest. For all of the above features, the measured and simulated elevation histories are in good quantitative agreement. Note that the constitutive coefficients of the theory are obtained from separate measurements of the transport relation \( J(Q, S) \), not calibrated on the basis of the measured channel response. The level of agreement recorded thus represents a faithful measure of the predictive power of the theory. Adjusting the coefficients based on the measured histories would improve the fit, but make the comparison less meaningful.

To check that the simulated and measured responses agree over a wider set of conditions than those of a single experimental run, Fig. 13 shows lake decay time obtained for 9 different runs. Based on the dimensional analysis outlined in Hsu and Capart [2008], the time to lake elimination \( t_3 \) should be governed by a relation of the form

\[
\frac{t_3 - t_2}{t_2 - t_1} = F\left( \frac{I_1}{J_0} \right),
\]

(24)

where \( t_1 \) and \( t_2 \) mark the beginning and end of lake growth, \( J_0 \) is the background bedload transport rate in the trunk river, and \( I_1 \) is the tributary influx during lake growth. Figure 13 shows the resulting curve, obtained by simulating the full lake life cycle for different ratios \( I_1 / J_0 \) of tributary sediment influx to trunk river sediment transit. According to the theory [Hsu and Capart, 2008], tributary-dammed lakes form only when this ratio exceeds the threshold \( I_1 / J_0 = 2 \). Otherwise, the tributary sediment influx is too weak to
force the formation of a lake, and causes instead a cuspatate river aggradation. This explains the zero intercept of the calculated curve at the value $I_1/J_0 = 2$. As seen in Fig. 13, the experimental results are in good agreement with the calculations. Again, we stress that this agreement is attained without calibrating simulation parameters against the measured channel responses. For simplified laboratory conditions, the theory and numerical scheme are thus checked to reliably predict the river and lake evolution.

6. Laonong river simulations

6.1. Parameter estimation

Using the model presented in the preceding sections, we now simulate the 2007 episode of lake formation and decay at the confluence of Laonong and Pu-tun-pu-nas Rivers. Although we were able to gather much information about this field event (Sections 2 and 3), there remain significant data gaps. Most importantly, the tributary sediment influx hydrograph $I(t)$ is unknown. The model therefore cannot be run simply in forward mode, based on completely known inputs. It must partly be used as an inversion tool, to determine unknown inputs from the available data. For this purpose, our approach is to reduce the unknowns to a minimal set of parameters, then adjust their values by trial and error to match model outputs with survey data.

Three categories of inputs are needed to model the trunk river response: a choice of initial conditions prior to lake formation; information on the river flow and its bedload transport capacity; and information on the sediment influx from the Pu-tun-pu-nas tributary. In the years preceding the lake event, ground photos indicate moderate, intermittent sediment supply from the tributary. This was sufficient to maintain a cusped slope break $S_{01} > S_{uv}$ of the trunk river at the tributary junction, but not enough to force the formation of a lake. For the initial conditions, we therefore assume that the Laonong river profile starts at time $t_0$ from the
piecewise linear profile of Eq. 17, with initial value \( z_t(0, t_0) = z_0 \) at the confluence, and constant slopes \( S_U \) and \( S_D \) upstream and downstream. We determine parameters \( z_0, S_U, \) and \( S_D \) by best fit from the profiles surveyed in August 2004 by the WRA.

Somewhat arbitrarily, we choose the date of May 1st, 2007, the beginning of the 2007 typhoon season, as the starting time \( t_0 = 0 \) of the simulations. Conditions of dynamic equilibrium appear to have prevailed at the confluence prior to the lake formation event. We therefore assume that the initial conditions of our simulations in May 2007, can be approximated by the profiles surveyed in August 2004. We could start simulations instead from the actual date of the 2004 survey, but this would only displace the problem, since we would then have to reconstruct the unknown history of the tributary sediment influx from August 2004 to May 2007.

Information on river flow is more complete. The water discharge hydrograph \( Q(t) \) is given by the reconstructed daily streamflow data at Hsing-huei Bridge (Section 3.2). For the channel width, we adopt the value \( B = 90 \) m, averaged from 20 transects retrieved from air photos, and checked against widths estimated from our surveyed cross sections. For the normal depth relation (Eq. 7), we adopt the rating curve coefficient \( C = 0.21 \) determined from the depth-discharge relation at Hsing-huei Bridge (Section 3.3). The only relationship that is poorly constrained is the bedload transport law \( J(Q, S) \), for which no in situ data are available. Earlier studies of river morphodynamics [Lane, 1955; Paola, 2000] suggest that the simple stream power relation \( J = KQS \) obtained by setting \( S_{\text{min}} = 0 \) in Eq. 8 is appropriate at field scales. In the absence of more detailed information, we adopt this assumption, and only the transport coefficient \( K \) remains to be determined. This coefficient is retained as a free parameter, to be calibrated against the observed river response.
The remaining crucial data gap concerns the time history of the tributary sediment influx, from May 2007 to February 2008. In the absence of any direct measurements or observations of the debris flows responsible for this influx, we adopt for the tributary forcing the simple functional form

\[
I(t) = \begin{cases} 
0, & 0 \leq t < t_1, \\
\kappa_1 Q(t), & t_1 \leq t < t_2, \\
0, & t_2 \leq t.
\end{cases}
\tag{25}
\]

This represents a single period of debris flow activity (possibly approximating the aggregate effect of multiple surges), lasting from time \( t_1 \) to time \( t_2 \). During this period, the tributary sediment influx is assumed proportional to the water discharge \( Q(t) \) in the Laonong river, taken as a proxy for the prevailing hydrological conditions. The corresponding dimensionless coefficient of proportionality \( \kappa_1 \) is unknown, as well as the times \( t_1 \) and \( t_2 \) marking the beginning and end of the sediment influx episode. Three additional free parameters, \( \kappa_1, t_1 \) and \( t_2 \), must therefore be adjusted by comparing model outputs with profile data at different stages of the lake life cycle.

6.2. Comparison of simulated and reconstructed long profiles

Five different river long profiles are available to constrain the simulations (Fig. 5B–C). First, the profile surveyed by the WRA in 2004 is used to approximate the initial trunk river profile on May 1, 2007. Secondly, the 2007 terrace riser profile surveyed in February 2008 is interpreted as a record of the sediment bed elevation of the trunk river at the time of maximum aggradation. This time is unknown, but coincides in our simulations with the time \( t_2 \) of termination of the tributary sediment influx. An additional partial profile is provided by the water surface of the decaying lake on September 26, 2007, estimated from ground photographs. Prior to the elimination of the lake, this is the only profile for which a precise
time stamp is available. Next, we interpret the elevations of slack water deposits along the 
banks of our surveyed cross sections as points along the high water profile of the last previous 
flood experienced by the river. This corresponds to the Krosa typhoon flood, which peaked on 
October 7, 2007. Finally, our survey acquired a detailed profile of the low-flow water line on 
February 22, 2008. The computations were performed with upstream and downstream 
boundaries at locations $x_u = -5$ km, $x_d = 5$ km, spatial step $\Delta x = 10$ m, and variable time 
step $\Delta t$ dependent on the evolving magnitude of the water discharge, subject to the stability 
constraint (Eq. 16).

As described above, four free parameters had to be calibrated by trial and error: transport 
coefficient $K$, tributary influx coefficient $\kappa_1$, and times $t_1$ and $t_2$ marking the beginning 
and end of the tributary influx episode. Because four long profiles are available for 
comparison (not counting the initial profile used to calibrate $S_u$, $S_d$, and $z_0$), more than 
enough information is available to constrain these free parameters. We determined their 
values by seeking the best fit between simulated and observed profiles. We find that 
parameters can be determined unambiguously, with a significant drop in fit quality if any of 
them departs from the retained values. Times $t_1$ and $t_2$ obtained by best fit confirm that the 
period of high tributary activity coincided with Typhoons Wutip and Sepat. The period starts 
with the arrival of Typhoon Wutip, on August 8, 2007, and ends before Typhoon Wipha. 
Simulations indicate that the tributary dam reached its maximum elevation on September 15, 
and that the lake vanished on October 23. Although the exact timing of the two events is 
unknown, these dates are consistent with the available field information (see Table 2).

Based on the above reconstruction and calibration, observed and simulated field profiles are 
plotted in Fig. 14. The profiles document a lake life cycle that is remarkably similar to the one
produced in the laboratory experiments (Fig. 11). Starting at time $t_0$ from an elevation profile that monotonously decreases down valley (Fig. 14A), the river bed responds to the tributary sediment influx by forming a temporary dam at the tributary junction. At the time $t_2$ of maximum aggradation (Fig. 14B), the dam crest reaches elevation $z_s^{\text{max}} = 624.5$ m, or about 20 m above the initial river bed elevation $z_0 = 603.75$ m. The tributary sediment influx which caused this dramatic aggradation is estimated to have lasted $t_2 - t_1 = 38$ days, during which the volume of sediment dumped by the tributary into the Laonong river can be evaluated from

$$ V_s = \int_{t_1}^{t_2} I(t) dt. \quad (26) $$

This amounts to approximately 980 000 m$^3$ of bed material (sediment + pore space), or about $1.6 \times 10^6$ tons of sediment (dry weight of the granular phase). At the time of maximum aggradation, the simulated lake reached a maximum length $L_{\text{max}} = 1.6$ km, double the length observed during lake decay on September 26, 2007.

Upon termination of the tributary sediment influx, the lake underwent a phase of decay and drainage, characterized by incision of the lake outlet and lowering of the lake water level. The state of partial lake decay observed on September 26 (Fig. 14C) corresponds to a water level which has dropped some 10 m below the maximum lake level. The lake appears to have survived at least until the peak of typhoon Krosa, on October 7, 2007 (Fig. 14D). Under the influence of the peak water discharge $Q = 380$ m$^3$ s$^{-1}$, close to the maximum discharge experienced during the simulation period, the water depth in running-water reaches is estimated to have attained 3.4 m on that date. By the time of our survey on February 22, 2008 (Fig. 14E), the lake had completely disappeared, and the river bed approximately returned back to its elevation prior to the episode.
Through successive stages of lake formation and decay, the simulated and observed profiles (Fig. 14) are in good agreement. At the price of four adjustable coefficients, the simulations successfully reproduce the observed river and lake evolution. The agreement generally holds both upstream and downstream of the confluence, for the entire lake life cycle. An exception is the river bed downstream of the confluence on February 22, 2008 (Fig. 14E). Over range $0 < x < 1000$ m, the surveyed profile lies approximately 3 m below the computed profile. Gravel mining activity observed in this area during our survey provides a possible explanation for this difference.

6.3. Elevation histories and stage hydrographs

To further describe the trunk river behavior, Fig. 15 shows simulated time histories of the sediment bed and water surface elevations at five different cross sections: one at the tributary junction, two upstream of the junction, and two downstream. The first upstream cross section coincides with the location of Hsing-huei Bridge, and the second downstream cross section coincides with the location of the planned Laonong Weir (see Fig. 5). The simulated time histories for the field event (Fig. 15) resemble in many ways those obtained for the laboratory experiments (Fig. 12). Downstream of the tributary junction, the bed elevation histories track the rise and fall of the tributary dam crest, subject to both delay and attenuation. Upstream of the junction, the sediment bed at Hsing-huei Bridge experienced a delayed rise, a period of constant elevation when drowned by the lake, then a sudden jump upon passage of the deltaic infill front. This evolution is transmitted, in delayed and attenuated form, to the second cross section further upstream, out of reach of the lake.

For the river sediment bed evolution (Fig. 15A), two differences with the laboratory experiments (Fig. 12A) can be noted. In the initial stages of the field case, the trunk river acts
to diffuse the slope break at the tributary junction, leading to some degradation of the bed prior to the tributary sediment influx. The second difference concerns the appearance of the curves, more irregular in the simulations of the field event. This is due to unsteady variations of the water discharge $Q(t)$, affecting both the bedload transport capacity of the trunk river (via Eq. 8), and the rate of tributary sediment influx (via Eq. 25). In our simulation of the field event, the water discharge controls the pace of alluvial diffusion, and acts as a proxy for the hydrological conditions controlling tributary sediment influx. In the experiments, by contrast, the water discharge and tributary influx (during the period of lake growth) were held constant, leading to simpler curves.

The effect of unsteady water discharge is clearest in Fig. 15B, which shows the simulated water surface elevations at the five cross sections. Far from the confluence, fluctuations in water depth are superposed on mildly varying bed elevation variations. The depth fluctuations take the form of asymmetric pulses (receding limb slower than rising limb), mirroring the shape of the water discharge hydrograph (Fig. 6). Closer to the tributary junction, the water depth fluctuations are superposed onto much greater swings in stage due to the evolution of the tributary dam and lake. The most characteristic response occurs upstream of the junction, at Hsing-huei Bridge, where the water stage becomes controlled by backwater effects when the cross section is drowned by the lake. Instead of following the local bed elevation, the water stage at the bridge rises and falls with the crest of the tributary dam, 1 km downstream. Due to tributary influence, stage variations driven by river bed aggradation and degradation can therefore greatly exceed those due to fluctuations of the water discharge.

No extended records of sediment bed or water surface elevation are available to verify these simulated histories, except at Hsing-huei Bridge. Daily water stage measurements are recorded there until the bridge deck was submerged on August 19, 2007. After the demise of
the bridge, two more data points can be added, one from photographs taken on September 26, 2007, and the other from surveyed slack water deposits attributed to the Krosa Typhoon flood peak of October 7, 2007. As shown in Fig. 15B, the simulated stage hydrograph at Hsing-huei Bridge exhibits excellent agreement with the measured data. In addition to river and lake long profiles, the simulation can therefore reproduce the complex water stage evolution at Hsing-huei Bridge.

7. Conclusions

In this work, we documented a field example of lake formation and decay, and used the episode to test a simplified mathematical description of river and lake evolution. Considering the simplicity of the description, a surprising degree of agreement was obtained, showing that the proposed equations can reproduce not only small-scale experiments, but also field scale processes. For this field application, it was found possible to calibrate the model in a straightforward way, check its results using additional data, and use the simulations to fill in some of the inevitable data gaps. By combining mathematical modeling and field observations, we were thus able to obtain information that neither of these approaches could produce on its own.

According to our analysis, Typhoons Wutip and Sepat caused a period of sustained debris flow activity in the Pu-tun-pu-nas River. This led to strong sediment influx into the Laonong River, exceeding the infill capacity of the trunk river and forcing the formation of a lake. During lake growth, the trunk river played a role in shaping the dam-forming deposit. Acting concurrently with tributary influx, streamflow past the dam crest tempered dam growth and diffused the aggradation towards downstream reaches. Upstream of the confluence, meanwhile, sediment transport in the trunk river was interrupted due to the backwater influence of the growing lake. Upon termination of the tributary influx, streamflow from
Typhoons Wipha and Krosa caused the lake to decay and vanish, by simultaneously incising the dam and infilling the depression. Through these various stages of the lake life cycle, good agreement is registered between observed and simulated profiles.

Associated with this sequence of events, simulated histories of river bed elevation and water stage show considerable complexity. The sediment bed evolution is driven by successive pulses of streamflow, and strongly perturbed by sediment influx from the tributary. The water stage, on the other hand, responds both to slow streambed evolution and to fast fluctuations in streamflow. Due to lake formation, the upstream water stage responds not only to local bed changes, but also to the distant evolution of the dam crest, propagated upstream by backwater influence. As a result, stage records at Hsing-huei Bridge would be impossible to interpret without knowledge of the local and global evolution of the river bed elevation. By taking into account the evolution of water and bed profiles along the entire valley segment, however, we find that the stage record can be accurately reproduced using a relatively simple theory.

Nevertheless, we should add the cautionary note that this level of agreement may not carry over to conditions distinct from those of semi-alluvial, steep-sloped Taiwan rivers, evolving due to typhoon floods. If lower flow discharges are of interest, in particular, partial transport conditions may govern the morphodynamic response of the river bed. In that case, it may be necessary to take into account spatial shear stress variations and the differing mobility of separate sediment size fractions. Likewise, for laterally unconfined river segments flowing over alluvial fan surfaces or within wide braiding plains, channel migrations and width variations may exert key influences. Both aspects are disregarded by the present theory.

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Table 1. Reconstructed timeline of the 2007 lake formation and decay episode at the confluence of the Laonong and Pu-tun-pu-nas Rivers, southern Taiwan.

<table>
<thead>
<tr>
<th>Date [year/month/day]</th>
<th>$t$ [day]</th>
<th>Event or observation</th>
<th>Origin of the information</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007/05/01</td>
<td>0</td>
<td>Beginning of the 2007 typhoon season</td>
<td>By convention</td>
</tr>
<tr>
<td>2007/08/08</td>
<td>99</td>
<td>Streamflow increase due to Typhoon Wutip</td>
<td>Streamflow at upstream and downstream stations</td>
</tr>
<tr>
<td>2007/08/08</td>
<td>99</td>
<td>Start of period of debris flow activity in Pu-tun-pu-nas River</td>
<td>Deduced from best fit of simulation results</td>
</tr>
<tr>
<td>2007/08/13</td>
<td>104</td>
<td>Peak discharge of Typhoon Wutip</td>
<td>Streamflow at upstream and downstream stations</td>
</tr>
<tr>
<td>2007/08/19</td>
<td>110</td>
<td>Peak discharge of Typhoon Sepat</td>
<td>Streamflow at upstream and downstream stations</td>
</tr>
<tr>
<td>2007/08/19</td>
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<td>Last stage record at Hsing-huei Bridge</td>
<td>Water Resources Agency</td>
</tr>
<tr>
<td>2007/09/15</td>
<td>137</td>
<td>End of period of debris flow activity in Pu-tun-pu-nas River</td>
<td>Deduced from best fit of simulation results</td>
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<td>2007/09/19</td>
<td>141</td>
<td>Peak discharge of Typhoon Wipha</td>
<td>Streamflow at upstream and downstream stations</td>
</tr>
<tr>
<td>2007/09/26</td>
<td>148</td>
<td>Decaying lake</td>
<td>Field site visit</td>
</tr>
<tr>
<td>2007/10/07</td>
<td>159</td>
<td>Peak discharge of Typhoon Krosa</td>
<td>Streamflow at upstream and downstream stations</td>
</tr>
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<td>2007/10/23</td>
<td>175</td>
<td>Lake disappearance</td>
<td>Simulated</td>
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<tr>
<td>2008/01/06</td>
<td>250</td>
<td>Drained lake bed</td>
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<tr>
<td>Trunk river channel</td>
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<td>90 m</td>
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<td>Confluence elevation $z_0$</td>
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<td>Upstream slope $S_U$</td>
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<td></td>
<td>Downstream slope $S_D$</td>
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<td>Normal depth $h_0$</td>
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<td>Rating curve coefficient $C$</td>
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<td>0.0025</td>
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<td>Influx starting time $t_1$</td>
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<td>Influx stopping time $t_2$</td>
<td>239 s</td>
<td>day 137</td>
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<td>Maximum dam elevation $z_{max}$</td>
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<td>Maximum lake length $L_{max}$</td>
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<td>1625 m</td>
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<td>Time of lake disappearance $t_3$</td>
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n.a.: not applicable
**Figure legends**

Figure 1. Schematic of the natural lake formed in August 2007 at the confluence of a steep tributary (Pu-tun-pu-nas River) with a semi-alluvial trunk river (Laonong River, southern Taiwan). The tributary drains a small watershed, highly prone to landslides and debris flows, which contributed large amounts of sediment to the trunk river. This caused confluence aggradation and the formation of tributary dam across the trunk river. Other episodes of massive sediment supply from this tributary have occurred in the past, as recorded by the high fan terraces bordering the confluence, and occurred again more recently due to record-breaking rains from Typhoon Morakot in August 2009. The geological cross-section is drawn after WRA (1997).

Figure 2. Setting of the field study: (a) Taiwan Island; (b) Laonong River Basin; (c) Confluence between the Laonong and Pu-tun-pu-nas Rivers. Faults on panel (a) drawn after Malavieille and Trullenque [2009]. On panel (c), terraces are drawn after Hsieh and Chyi [2010], and the bank classification is based on air photos and ground observations.

Figure 3. Formation and decay of a lake at the confluence of the Pu-tun-pu-nas River with the Laonong River: (a) tributary fan on June 10, 2005; (b) raised fan surface on September 26, 2007; (c) tributary-dammed lake viewed from upstream of the confluence on September 26, 2007; (d) drained lake bed on January 6, 2008; (e) Hsing-huei Bridge, 1 km upstream of the confluence, on October 24, 2006 (Source: Water Resources Agency); (f) the bridge site on September 26, 2007. The bridge deck has been washed away, but the central pile and lateral abutments remain.

Figure 4. Panoramic views of the Pu-tun-pu-nas tributary dam at various stages of degradation,
viewed from the opposite bank of the Laonong River: (a) during lake decay on September 26, 2007; (b) after complete drainage of the lake on January 6, 2008; (c) after typhoon Kalmaegi, on July 23, 2008. Flow in the trunk river is from right to left.

Figure 5. Topographic data surveyed before and after the 2007 episode of lake formation and decay: (a) plan view of the survey lines; (b) long profiles of the trunk river, tributary terrace riser, and slackwater deposits; (c) close-up of the lake site with reconstructed water levels; (d) cross-section of the trunk river and tributary long profile at the confluence. Wood samples retrieved from the exposed flank of the elevated fan terrace indicate ages between 3000 and 7000 Before Present [from Hsieh and Chyi, 2010]. Similar in geometry to the new terrace created in 2007, this ancient terrace may be the remnant of a large tributary dam. The left bank of the trunk river opposite the tributary is composed of exposed bedrock.

Figure 6. Streamflow at Hsing-huei Bridge from August 2004 to December 2007. Black silhouette: daily streamflow estimated by averaging signals from upstream and downstream stations. Exceptions are March to September 2005 (only the upstream signal is used), and October 2005 to May 2006 (only the downstream signal is used). Triangles: discharge measurements at Hsing-huei Bridge. Peaks labels are typhoon names, and time $t_0 = 0$ corresponds to the beginning of the 2007 typhoon season (May 1st, 2007).

Figure 7. Variation of stage and depth with discharge: (a) absence of one-to-one relation between measurements of stage and discharge at Hsing-huei Bridge; (b) relation between measured stage $z_w$ and estimated daily discharge $Q$ at Hsing-huei Bridge: on a plot of $z_w$ versus $Q^{2/3}$, short term variations trace parallel segments of approximately constant slope $a$; (c) reconstructed depth-discharge relation.
Figure 8. Predicted profiles for laboratory example of lake growth and decay: (a) computed stream bed (solid lines) and water stage profiles (dashed lines) at times $t = 0, 46, 103, 157, 214,$ and $239$ s (lake growth); (b) comparison of the calculated stream bed profiles of panel (a) (thin solid lines) with the analytical solution of Hsu and Capart [2008] (bold dashes), in normalized coordinates highlighting self-similarity; (c) computed stream bed (solid lines) and stage profiles (dashed lines) at times $t = 239, 268, 298, 332, 365,$ and $393$ s (lake decay and elimination), with the initial stream bed profile ($t = 0$) added for reference. From here on flow is from left to right.

Figure 9. Laboratory set-up used for the experiments of Hsu [2007].

Figure 10. Laboratory example of lake growth and decay, from the experiments of Hsu [2007]: (a)-(e) photographs at times $t = 0, 103, 239, 298,$ and $393$ s. Tributary sediment influx was started at time $t_i = 0$ (photo a), and stopped at time $t_i = 239$ s (photo c).

Figure 11. Measured profiles for laboratory example of lake growth and decay: (a) stream bed (solid lines) and water stage profiles (dashed lines) at times $t = 0, 46, 103, 157, 214,$ and $239$ s (lake growth); (b) stream bed (solid lines) and stage profiles (dashed lines) at times $t = 239, 268, 298, 332, 365,$ and $393$ s (lake decay and elimination), with the initial stream bed profile ($t = 0$) added for reference.

Figure 12. Time evolution of stream bed elevation and water stage for experimental example of lake growth and decay: (a) predicted (solid lines) and measured (circles connected by dashed lines) bed elevation hydrographs at positions $x = -500, -250, 0, 250,$ and $500$ mm; (b) predicted and measured water stage hydrographs at the same positions.
Figure 13. Time to lake elimination (normalized by the duration of lake growth), plotted against the ratio of tributary sediment influx $I_1$ (during lake growth) to trunk river sediment transit $J_0$. Solid line: theoretical prediction (computed by finite differences); triangles: experimental measurements for 9 different laboratory runs. Higher rates of tributary sediment influx (or longer lasting lake growth) create larger lakes, which survive longer after tributary influx is terminated.

Figure 14. Comparison of simulated and surveyed trunk river long profiles at different stages of the lake life cycle: (a) before lake formation ($t = 0$); (b) at maximum dam aggradation ($t = 137$ days); (c) during lake decay ($t = 148$ days); (d) at peak discharge of Typhoon Krosa ($t = 159$ days); (e) after disappearance of the lake ($t = 297$ days). See Table 1 for correspondence between simulation times $t$ and calendar dates.

Figure 15. Simulated time evolution of bed elevation and water stage at different cross-sections of the trunk river (solid lines), located respectively at positions $x = -1900, -950, 0, 700, 1400$ m relative to the confluence: (a) bed elevation; (b) water stage. Circles: stage measurements at Hsing-huei Bridge.
Figure 05
Figure 07
Figure 10
Figure 12
Figure 13
Figure 15

[Graph A] Shows a comparison of water level (m) over time (t day) with different scenarios. The graph indicates variations from 580 to 630 meters.

[Graph B] Displays a more detailed analysis with additional markers and fluctuation patterns, showing similar time intervals for comparison.