Response of hyperpycnal deltas to a steady rise in base level

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ABSTRACT: The subaqueous evolution of alluvial deltas is sometimes driven by turbidity currents plunging along the delta foresets. In this paper, we examine how such hyperpycnal deltas evolve under conditions of steadily rising base level, associated either with a gradually filling reservoir, or with a rising sea level. The hyperpycnal delta evolution is envisioned as a one-dimensional diffusion process, with different diffusivities acting along the topset and foreset, and the resulting equation is solved by finite differences. Computations are first validated against analytical solutions derived earlier for the case of constant base level. Numerical simulations for the case of rising base level are then presented, and compared with small-scale laboratory experiments. The numerical, analytical and experimental results are found to be in good agreement with each other, and exhibit various features of interest. Like deltas evolving under homopycnal inflows, hyperpycnal deltas starting from a uniform slope first prograde, then retreat under the influence of a rising base level. Unlike the homopycnal case, however, a rising base level can cause erosion along the upper face of hyperpycnal foresets. We hypothesize that this could provide a mechanism for the incision of near-shore underwater canyons.

1 INTRODUCTION

Over geological or engineering time scales, variations in sea and lake levels can exert a significant influence on depositional patterns at river mouths. Whereas deltas tend to steadily prograde into the receiving basin when the base level stays constant, variable base levels can lead to more complex evolutions of deltaic shorelines. Even for a relatively simple, steady rise in base level, Muto and Steel (1992) and Muto (2001) have shown using a conceptual model and laboratory experiments that the delta response can feature multiple phases of evolution. Starting from a basement of constant inclination, a steady water and sediment supply will at first lead to a delta that progrades seaward. At some point, the depositional build-up of the delta foreset continues, but the shoreline starts to retreat landwards. This is followed by a phase of auto-retreat, during which the delta foreset is starved of further supply, and the shoreline continues its landward retreat.

In this model and experiments, the delta response was examined for a Gilbert-type delta morphology (Gilbert, 1890) devoid of bottomset beds, with foreset beds of constant inclination along which deposits avalanche. This is the typical morphology expected for hypo- or homopycnal conditions at the river mouth, in which the density of the river inflow is lower or equal to the density of the receiving water. There are however many river mouths where, at least under certain conditions, the river inflow is denser than the receiving water. This occurs when river flow laden with fine sediment particles enters a fresh water lake, or when a heavily sediment-laden inflow intrudes into sea water. In that case, the river flow can feed density currents plunging along the bottom of the receiving ambient. Such hyperpycnal inflows influence the morphology and evolution of the delta, typically leading to foresets of mild inclination (Kostic et al., 2002) and upwards concavity (Adams et al., 2001) different from those of Gilbert-type deltas.

In Lai and Capart (2007), we proposed a model for the evolution of hyperpycnal deltas, and applied it to laboratory-scale deltas prograding into basins of constant water level. Our objective in the present work is to explore the response of such deltas when the base level evolves. Specifically, we examine the response of hyperpycnal deltas to a steadily rising base level, associated with sea level rise or with a gradually filling reservoir (Fig. 1). In the following sections, we report simulations conducted by solving a two-diffusion model using finite differences. We then compare simulated results with small-scale laboratory experiments. Finally, we discuss some possible implications of the results obtained.

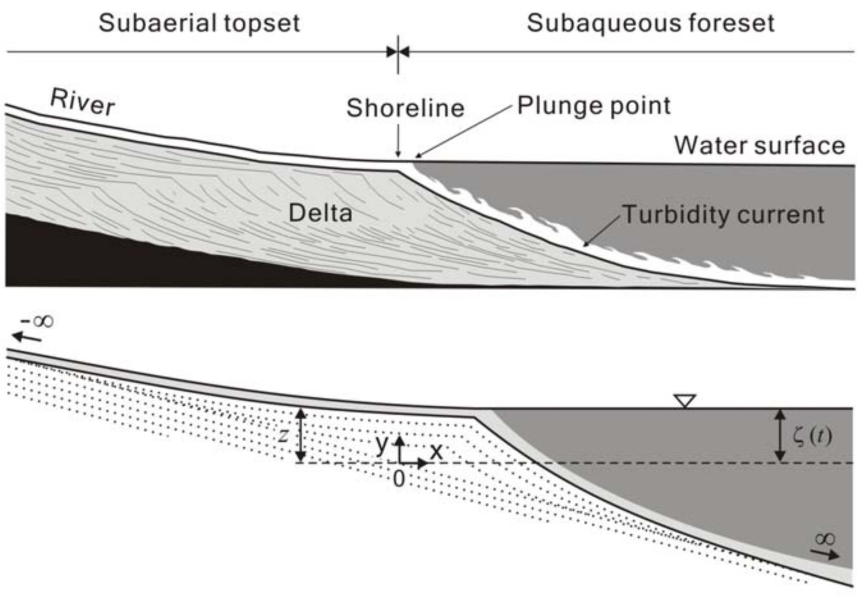

Figure 1. Definition of a hyperpycnal delta and mathematical description.

2 GOVERNING EQUATIONS

The hyperpycnal delta evolution is assumed to be governed by an Exner equation for the bed profile $z(x,t)$:

$$\frac{\partial z}{\partial t} + \frac{\partial q}{\partial x} = 0\,, \tag{1}$$

where q is the bedload transport rate driven by stream flow along the subaerial delta topset, and by density current along the subaqueous foreset (see Fig. 1):

$$q = \begin{cases} D_1 \max(-\partial z/\partial x - S_{\min}, 0), & z > \zeta\,, \\ D_2 \max(-\partial z/\partial x - S_{\min}, 0), & z < \zeta\,. \end{cases} \tag{2}$$

Here $S_{\min}$ is an inclination threshold below which no bedload transport occurs, D_1 and D_2 are diffusivities which take different strengths along the topset and foreset, and $\zeta(t)$ is the lake level evolving under the prescribed history

$$\zeta = Wt\,. \tag{3}$$

The initial and boundary conditions are

$$z = -Sx\,, \quad t = 0\,, \tag{4}$$

$$\frac{\partial z}{\partial x} = -S\,, \quad x \to \pm\infty\,, \tag{5}$$

where S is the initial bed slope. More details about the assumptions and derivation leading to these equations are provided in Lai and Capart (2007).

3 NUMERICAL MODEL

To treat general initial data and boundary conditions, the above two-diffusion governing equations must be solved by numerical means. For this purpose, we adopt a straightforward finite difference scheme that can be briefly described as follows. The spatial domain is first discretized into a series of equally sized cells, with bed elevations $z_i(t) = z(x_i, t)$ sampled at the centers of the cells

$$x_i = x_{\mathrm{L}} + (i - \tfrac{1}{2})\Delta x\,, \tag{6}$$

Starting from prescribed initial data, the bed elevation is advanced in time using the explicit finite volume statement

$$z_i(t+\Delta t) = z_i(t) + \frac{\Delta t}{\Delta x}\{q_{i-1/2}(t) + q_{i+1/2}(t)\}\,, \tag{7}$$

where $q_{i+1/2}$ is the bedload flux between cell i and cell i+1. This flux is calculated using the following approximation

$$q_{i+1/2} = \overline{D}_{i+1/2} \max(S_{i+1/2} - \overline{S}_{\min,i+1/2}, 0)\,, \tag{8}$$

where

$$\overline{D}_{i+1/2} = \tfrac{1}{2}(D_i + D_{i+1})\,, \tag{9}$$

$$S_{i+1/2} = \frac{z_i - z_{i+1}}{\Delta x}\,, \tag{10}$$

$$\overline{S}_{\min,i+1/2} = \tfrac{1}{2}(S_{\min,i} + S_{\min,i+1}). \tag{11}$$

The scheme has various advantages. The finite volume statement makes the scheme shock-capturing, allowing the shoreline slope break and diffusivity jump to be treated automatically inside the domain.

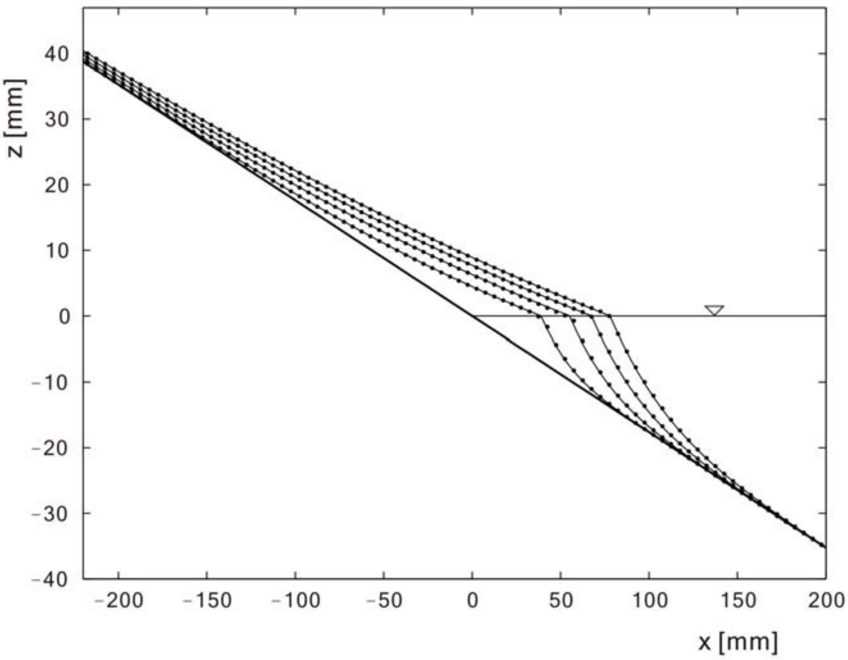

Figure 2. Comparison of the numerical (dots) and analytical results (lines), for a delta prograding into a lake of constant level.

Also, conservation of bed sediment is satisfied exactly, and the scheme is very easy to code. Because the time-stepping is explicit, stability requires

$$\Delta t \le \frac{(\Delta x)^2}{2\max(D_i)}, \tag{12}$$

and this limits the spatial resolution that can be attained at a reasonable computational cost. In one spatial dimension, however, this limitation is not too severe, and good accuracy is attained even with relatively coarse discretizations. The treatment of boundary conditions is standard and described in detail in Lai (2006).

Before applying the scheme to problems in which the base level varies, results for a constant base level can be checked against analytical results. Derived in Capart and Lai (2007), the analytical solution describes the self-similar evolution of a delta prograding into a lake of constant water level, starting from a bed of constant slope extending infinitely far upstream and downstream. In Fig. 2, numerical and analytical profiles for this case are plotted together and show excellent agreement.

4 RISING BASE LEVEL SIMULATIONS

Simulation results can now be presented for the case of a steadily rising base level (Fig. 3). The conditions chosen are derived from the laboratory experiments discussed in the following section. The starting profile is given a constant slope of inclination $S = \tan 10^\circ$. At time t = 0, sediment and dense river inflow is supplied upstream, and the lake level starts to rise at constant rate $W = 0.26$ mm/s. The diffusivities along the subaerial topset and subaqueous foreset are given values $D_1 = 491$ mm^2/s and $D_2 = 9$ mm^2/s, respectively. Finally, the inclination threshold $S_{\min}$ is set equal to $0.4S$.

In its initial stages (Fig. 3A), the delta morphology for the case of rising base level resembles that for constant base level presented in Fig. 2. Both the topset and foreset profiles are concave upwards throughout their lengths, reflecting depositional conditions. Unlike the avalanching foresets of Gilbert-type deltas, note that the foreset profiles of the present hyperpycnal deltas connect smoothly with the bathymetry towards the deep end of the basin. Like the constant base level case, the shoreline initially advances lakeward. The *x*-*z* path of the shoreline break, however, curves upwards in order to follow the lake level evolution. This resembles the initial evolution of the variable sea level Gilbert delta experiments of Muto (2001), reproduced using numerical computations by Parker and Muto (2003).

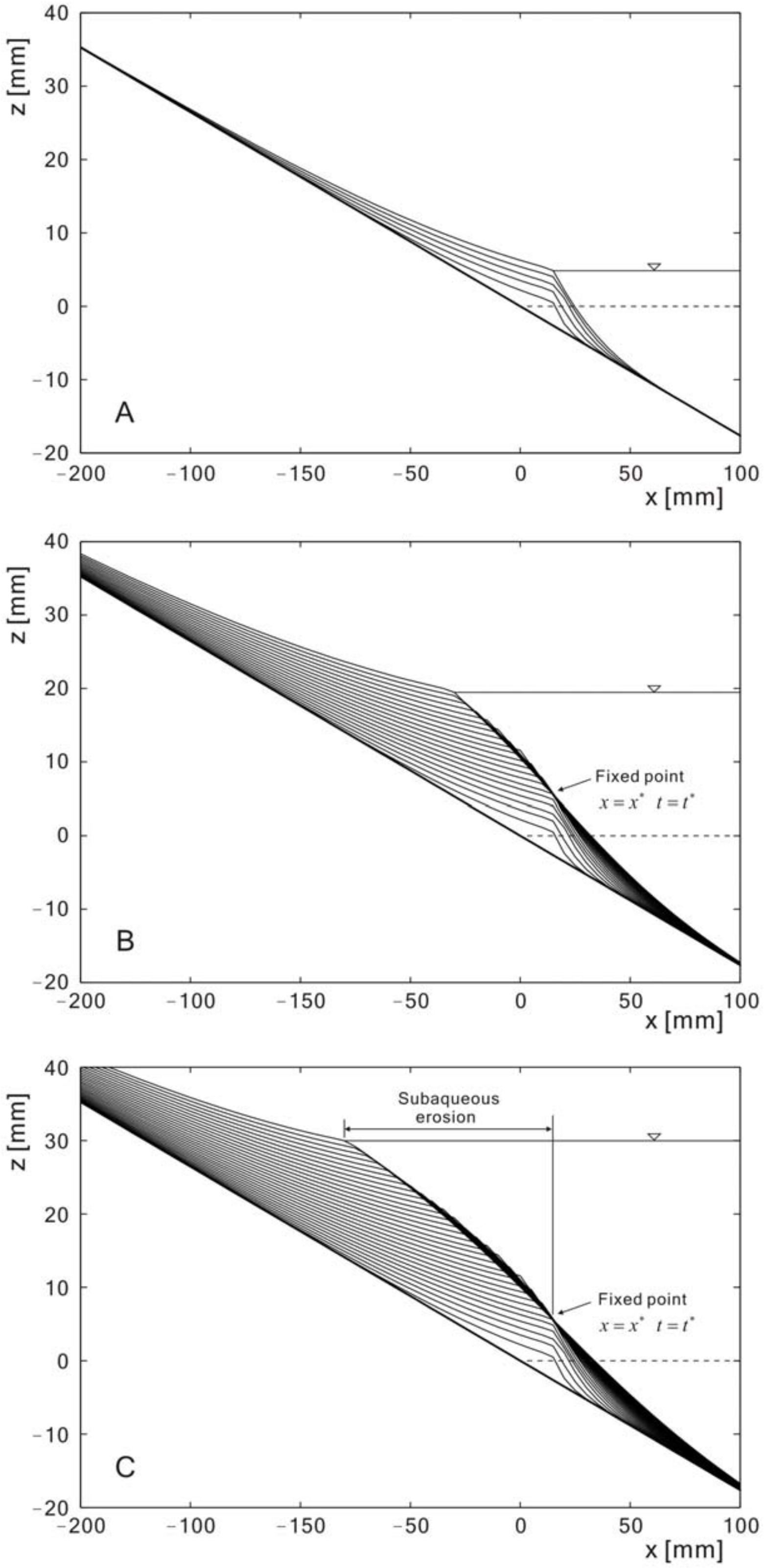

Figure 3. Simulated profiles for a small-scale hyperpycnal delta responding to a steadily rising base level. A, t = 50 s; B, t = 200 s; C, t = 300 s.

At later stages (Figs 3B and 3C), the effects of lake rise on the delta morphology and shoreline path become much stronger. The topset sediment supply is unable to keep up with the increase in accommodation space due to the lake level rise, and the shoreline begins to retreat landward. The resulting shoreline path in the *x-z* plane curves landward as it tracks the rising lake level. It is useful to contrast these results with the experiments of Muto (2001) on Gilbert-type deltas subject to rapid sea level rise (see also the analysis by Muto and Steel, 1992, and Parker and Muto, 2003). In these experiments, an initial progradation and ensuing retreat were also observed, with a similarly curved shoreline path. At

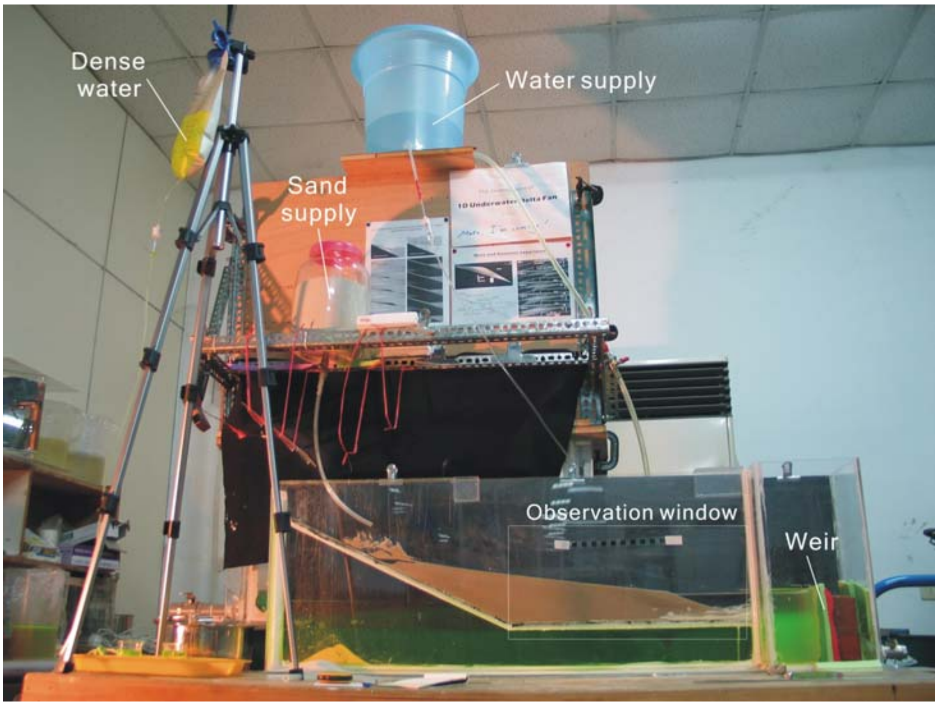

Figure 4. Experimental set-up used for the small-scale laboratory experiments.

some point, however, the backward inclination of the shoreline path exceeds the angle of repose of the Gilbert-type foreset. At this point, called "auto-break" by Muto and co-workers, the sediment supply at the delta front is completely stopped, and the shoreline "auto-retreats". Beyond this point, the topset continues to aggrade, but the delta front no longer evolves once it is submerged by the rising lake level.

In the present hyperpycnal delta simulations (Figs 3B and 3C), a different response of the delta front is obtained at later stages of the evolution. Because of the continuous geomorphic action of the density underflow, the foreset evolution does not stop once the delta front loses its upstream sediment supply. Instead, incision occurs along the proximal portion of the foreset (closest to the shoreline), eroding previously deposited sediments. These sediments are then carried downstream by the density flow, supplying the distal portion of the foreset (towards the deep end of the lake) which continues to aggrade. This type of response of the delta front to rapidly rising sea level was earlier characterized by Jordan and Flemings (1991), based on their two-diffusion computations of foreland basin evolution.

An intriguing feature of our simulations, which does not seem to have been reported previously, is the apparent existence of a fixed point along the foreset, marking a boundary between zones of erosion and deposition. Under continued sea level rise, the zone of incision along the proximal portion of the foreset continously expands landward, and the zone of deposition along the distal end of the foreset continuously progresses lakeward, but the boundary between the two remains stationary. This is surprising because there is no prescribed control at that location, and nothing would seem to prevent the migration of this internal boundary.

5 COMPARISON WITH EXPERIMENTS

To test the computations described in the previous sections, small-scale laboratory experiments were conducted at the Hydrotech Research Institute of National Taiwan University. The apparatus used for this purpose, illustrated in Fig. 4, is a small-scale version of the experimental set-up developed by Garcia (1993) for the study of turbidity currents. Flow takes place in a narrow flume having the following dimensions: 100 cm length, 1 cm width and 40 cm height. Upstream, a steady brine solution representing dense river inflow is provided by an elevated medical pouch. Next to the liquid supply, dry sediment is fed using a silo placed above the flume. The sediment used Ottawa sand (median diameter of 0.17 mm). The downstream end of the flume features a receiving tank with the following dimensions: 20 cm long, 20 cm width and 40 cm height. To make a rising base level, water supply is fed directly into the receiving tank at the highest part above the flume. The discharge of the water supply can be adjusted by a valve to control the base level rising speed.

Experimental results are shown in Fig. 5, for the case of a hyperpycnal delta evolving under a steadily rising base level. Initially (bottom panel of Fig. 5), the sand bed has constant inclination. When the inflow is turned on, a thin brine current (mixed with

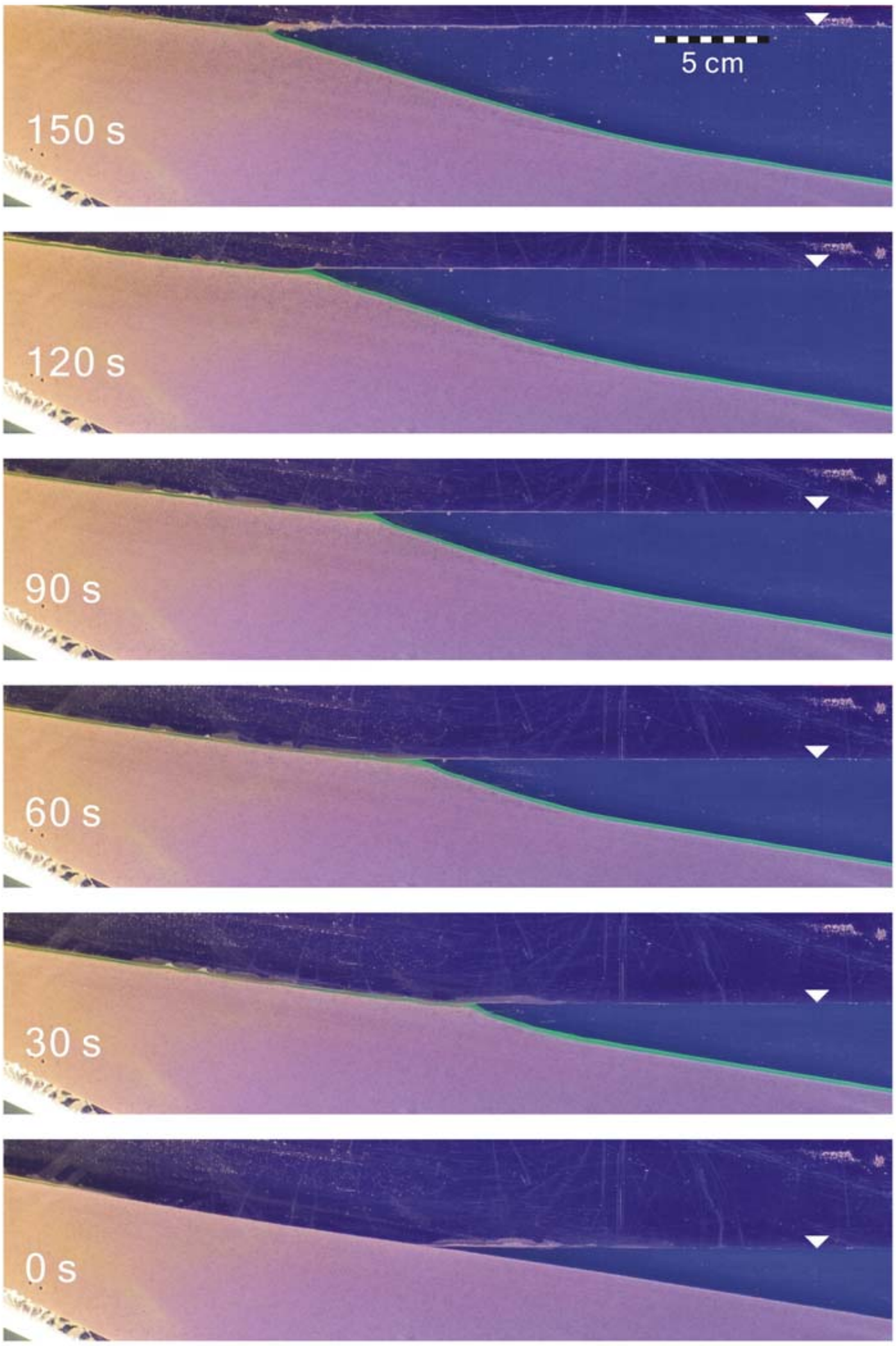

Figure 5. Series of experimental photographs depicting the progradation and subsequent retreat of a hyperpycnal delta subject to a steady rise in base level.

fluorescent dye) propagates along the bed surface, and plunges into the ambient at the shoreline. To record the process, a camera was used to take photographs every 5 seconds. This time-lapse photography approach is inspired from the approach used by Muto (2001) and Muto and Swenson (2005) for their small-scale Gilbert delta experiments. More detail about the laboratory set-up and image analysis procedures used can be found in Lai (2006).

The experimental results documented by these photographs are in qualitative agreement with the simulations discussed in the previous section. Due to dense river inflow, both the subaerial topset and subaqueous foreset are initially concave upward. As the lake level steadily rises, the shoreline first advances lakeward, then retreats landward. At some point, the concavity of the upper portion of the foreset reverses, becoming concave downwards, reflecting a switch from deposition to incision.

In Fig. 6, measured profiles for rising base level are contrasted with profiles for constant base level reported earlier in Lai and Capart (2007). A comparison between measured and simulated profiles is presented in Fig. 7. The measured and calibrated parameters for this run are: brine density = 1.20 g/ml, brine supply rate = 2.2 ml/s, sand supply rate = 0.48 g/s, rate of lake level rise = 0.26 mm/s and initial inclination = 10 degrees. In the numerical com-

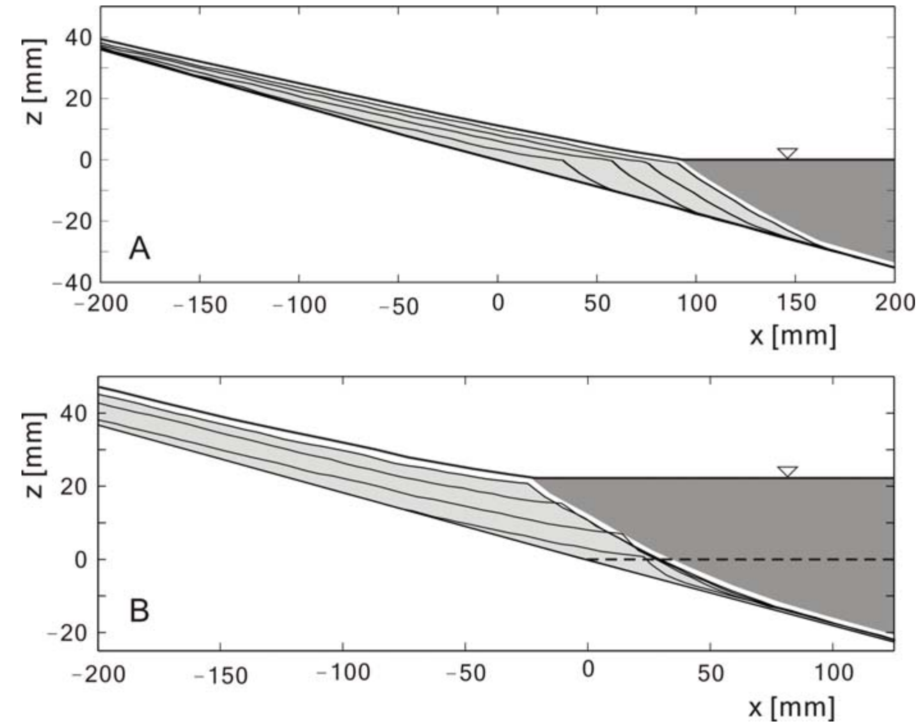

Figure 6. Measured long profile evolutions for two hyperpycnal delta experiments: A, constant base level (profiles at time $t = 20$, 65, 125, 200 s); B, rising base level (profiles at time $t = 10$, 35, 65, 90 s). Dashed line shows initial water level in the lake.

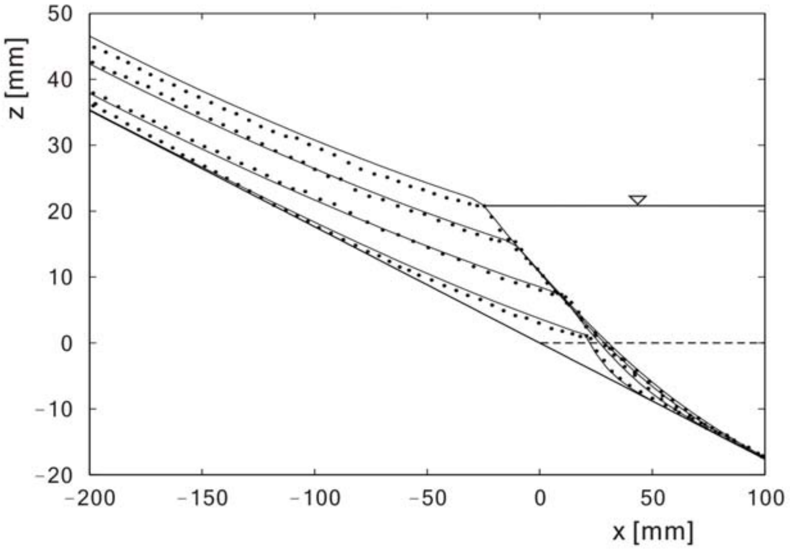

Figure 7. Comparison between the measured experimental results (dots) and numerical computations (lines).

putations, constant influx boundary conditions are applied at the actual sand supply position a finite distance upstream of the lake. The experimental results and numerical computation are in good agreement.

To further document and contrast the two runs depicted in Figure 6, shoreline position histories $s(t)$ are depicted in Fig. 8. Obtained under constant base level, the shoreline in case A advances monotonously lakeward. In case B, by contrast, obtained under rising lake level, the shoreline advances lakeward at first, then retreats landward, eventually (beyond t = 50 sec) retreating upstream of its starting point. For both cases, the shoreline position history is reasonably well reproduced by the calibrated analytical (case A) and numerical computations (case B).

6 CONCLUSIONS

In the present work, we have used simulations and experiments to examine the response of hyperpycnal deltas to a steady rise in base level. Like Gilbert-type deltas under similar conditions, the shoreline of hyperpycnal deltas first advances lakeward, then retreats landward as the base level rises. Unlike Gil-

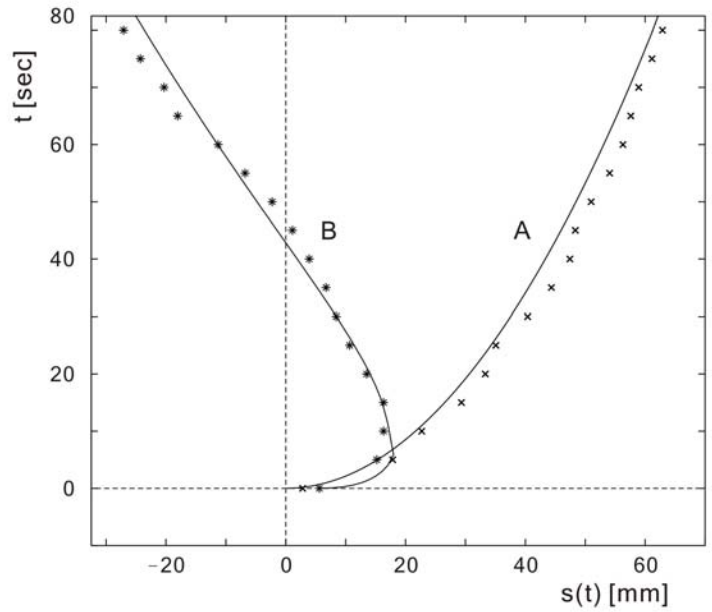

Figure 8. Comparison of analytical or numerical results (lines) and experimental results (symbols) for the time-varying shoreline position of hyperpycnal deltas. A, constant base level; B, steadily rising base level.

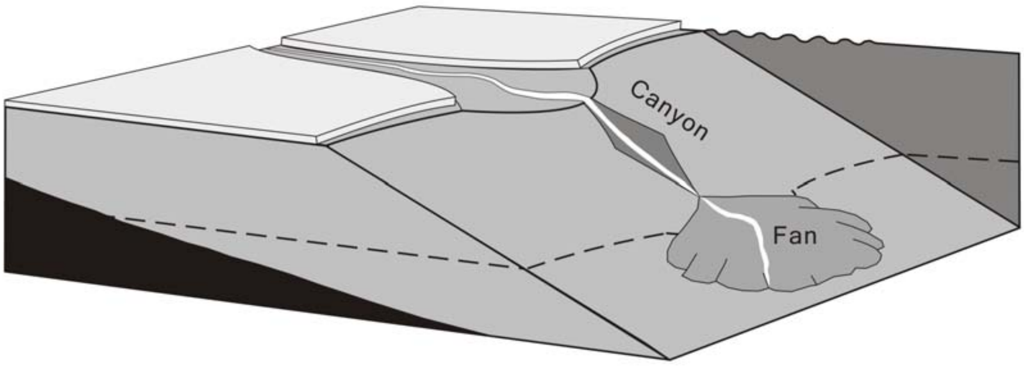

Figure 9. Sketch of a hypothetical mechanism for the incision of near-shore underwater canyons under steadily rising base level, due to the geomorphic action of hyperpycnal flows along the subaqueous foresets. The dashed line represents the initial base level.

bert-type deltas, however, the foresets of hyperpycnal deltas remain active throughout this evolution, with incision occurring in the proximal portion of the subaqueous deltas at late stages. As illustrated in Fig. 9, we hypothesize that this could provide a mechanism for the incision of near-shore underwater canyons.

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